Slope Stability Assessment by h-Method

William E Hodge

ABSTRACT

This article attempts to make the case that there is urgent need to include seepage forces into slope stability analysis equations wherever steady state seepage is involved in an earthfill. Towards that end a hydrodynamic formulation (h-Method) is proposed here in order to facilitate entering the relevant hydraulic information into what is otherwise seen as a mainly soil mechanics matter. A new stability analysis equation is offered which can take account of forces related to steady state seepage, or, at the operator's discretion, to ignore them. Using this mechanism it is possible to make a direct comparison between the Factors of Safety evaluated in particular cases where the seepage forces are taken into account, as opposed to the results when these are ignored. The equation is justified in detail and applied to four geotechnical variations of a safe neutral earthdam section. The results of this numerical experiment shows that ignoring seepage leads to computed Factor of Safety of up to twice those derived from their counterparts where seepage forces are acknowledged. It is therefore concluded that as things stand at the moment that it is possible that some engineered earthfills are much less stable than the designers appreciate, and this source of weakness may well account for some recent failures, particularly in the case of tailings dams.

Article 2 . . .

Slope Stability Assessment by h-Method

INTRODUCTION

The purpose of this article is to demonstrate that seepage forces must be included in assessing the slope stability of earthfill embankments designed to retain water, and then to provide a means for doing so.

What is presented here is a "start from scratch" examination of the forces at play within an earthfill embankment affected by seepage water. It is referred to herein as the h-Method, where "h" stands for hydrodynamic. This entailed creating a formulation for slope stability wherein terms for steady state seepage force components have an active place, and where these could be activated or not, in order to ascertain what effect they made.

The h-Method, rather than quoting independent variables in terms of stresses or pressures, works in terms of forces since vectors disclose both magnitude and direction of action. This allows them to be drawn and resolved as a force polygon where this graphical representation of forces helps to ensure that all vectors are apparent. And once the polygon closes some confidence is warranted that the approach is right.

This new stability analysis method was used on a single embankment geometry, however, with two foundation types (frictional and cohesive) and two drainage control measures (none and chimney drain), each with the seepage function *on* and *off*. The results of these eight computations are compiled for direct comparison and discussion.

SEEPAGE FORCE EVIDENCE

This photograph was taken through the glass wall of a water filled test tank, (Ref. 1).

The water level is indicated by the hand-drawn arrow near the upper right-hand corner. Clean sand was placed loosely within a cylindrical latex membrane. A drainage layer of fine crushed rock (the lighter color) underlay the sandfill. This drain was vented to the atmosphere by a vertical outlet. Then, after submergence within the tank, the membrane was gradually pulled up to expose the sandfill directly to the surrounding water. The dry (subaerial) angle of repose of this sand is 34°. Here it

may be seen to stand with a vertical side slope underwater. The reason for this counterintuitive behavior is the creation of an inward hydraulic gradient due to the differential radial water pressures, in other words, a phreatic surface inclined from the sand face to the inner drain.

Figure 1

Obviously, this real and powerful phenomenon ought to be made part of stability analyses computations.

PHREATIC SURFACE

On the downstream side of a water retaining earthfill there will be what is called a phreatic surface. Its position and shape is of primary importance when it comes to assessing the stability of that slope. This surface is the demarcation between several geotechnical phenomena:

a. it is the lowest level at which atmospheric pressure can exist in the pore spaces of the soil;
b. it is the boundary between the dry/moist soils above and the saturated ground beneath; and,

c. the pore water anywhere deeper down is ruled by the laws of hydrodynamics as they apply to steady state seepage.

These are the hydrostatic attributes of this surface and allows the following fact to be noted:

The effect of saturation on the ground is a purely gravitational matter and entails no lateral forces such as might influence slope stability. This can be established simply by considering a virtual column of solid particles immersed in water. By measuring the water pressure at the base of such a mixture it will be found equal to that which would have been measured if there had been only water and no solids, in other words in physical terms, the presence of solids has been treated in terms of pore water pressure as if their bulk had the same unit weight as water. So to balance the books the effective weight of the solids need to be reduced from saturated unit weight to buoyant unit weight. That is all.

It is quite a different matter when the hydrodynamic implications of the phreatic surface is recognized:

It is the uppermost flow line of a seepage net that has the reservoir level as its maximum potential, and tailwater level as its minimum potential. This network of pressures levels and streamlines allows the quantity, and rate, of water loss to be computed. This surface is also coincident with the locus of *epwp* (excess pore water pressure) with respect to tailwater level, as defined in Article 1 of this series. As such it is a measure of the potential energy available to the pore water within its precinct to work its way out of its soil confinement, and drain away from the embankment.

The point is that while the saturation of the downstream shell beneath the phreatic surface requires that the soil involved must be considered buoyant , this is quite a separate thing from the consequences of resisting the forces attending seepage.

COMPUTATIONAL PROCEDURE / METHODOLOGY

This new approach to assessing the stability of slopes is labeled the hydrodynamic method (h-Method). It differs from existing methods in that it recognizes, and deals with, the hydraulic energy naturally dissipated within a water retaining embankment.

As is common practice, the “method of slices” is employed here as a convenient means of distributing effective overburden weights to underlying sections of the trial failure surface. Since the boundaries of these imaginary slices have no real existence, no consideration is given to inter-slice forces.

Rather than adopting the practice of using the scalar quantities "pressures and stresses" to quantify physical magnitudes, here these are quoted as "forces" since vectors better inform the investigator by providing both magnitude and direction.

Vectors can be drawn and resolved as a force polygon. Such a graphical representation ensures that all vectors are apparent. The closing of the polygon is taken to be evidence that all elements of force relevant its formation are accounted for and appropriate. In consequence, examination of forces, as opposed to pressures or stresses, is a standard rule in the h-Method of analysis.

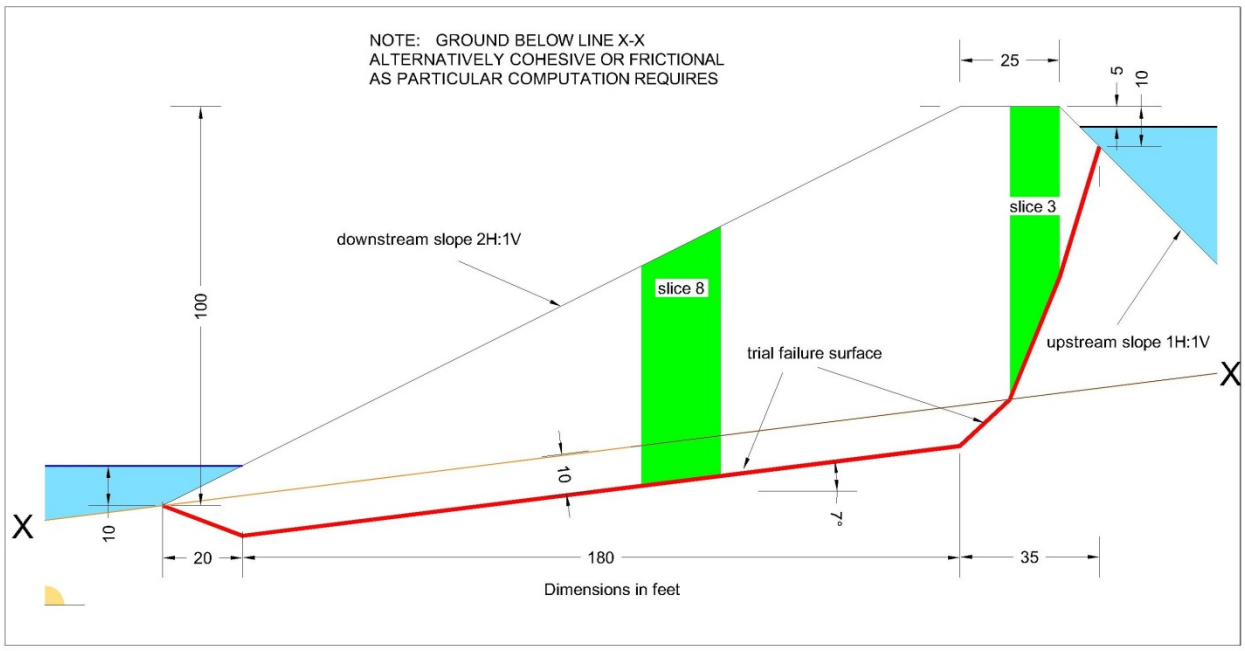

Figure 2

Figure 2 shows the slope geometry which was selected as the embankment section to be used as an example herein. It is a granular fill with 2H:1V outside slope inclination. The angle of shear resistance for the granular soil is set at φ = 35°.

The Factor of Safety of such a dry downstream slope, unaffected by other influences such as seepage or scouring, can be estimated at 3.3 ($\tan^{-1} 35^\circ / \tan^{-1} 1 \div 2$), a quite secure value.

In order to simplify subsequent interpretation it was established by trial and error, that for this geometry, and dry materials, the equivalent strength for cohesive soils would be 4,200 lb/ft^2. Consequently, these are the strength parameters used in the slope analyses reported here.

The head difference between reservoir level and tailwater is set at 85 feet; this is the source of hydrodynamic energy which results in seepage flow through the soils.

Evaluating Component Forces

The foot-pound system of units is used here. Following convention, volumes are taken as being numerically equal to surface areas on the understanding that thicknesses are of unit width/depth. The void ratio given to the granular fill is 0.2, whereas 0.5 was assigned to the cohesive soil.

Figures 3 and 4 show the physical conditions associated with Slices 3 and 8, respectively. There are two sketches on each figure: The one on the left side depicts the solid soil-structure forces and the pressures from which these are derived; the one on the right deals with the liquid phase - pore water. These two intimately related phases of the soil will be examined separately before combining their influence on stability. First, the solid phase:

Solids of soil-structure

The scale shown on the next three drawings (Figure 3, 4 and 5) are in terms of drawing units (du) in which 1 du therein represents: 1 foot length, or 100 lbs/sq ft pressure, or 2,000 lbs force. In Figure 6 the scale is 1 du represents 1,000 lbs force.

Here the phreatic surface is used to discriminate between the moist soil above and the saturated soil below. The moisture content above is set at the equivalent of 5% saturation, see Ref. 2. Soils below the phreatic line are considered fully saturated, and consequently exert only their buoyant weight on the soil-structure beneath.

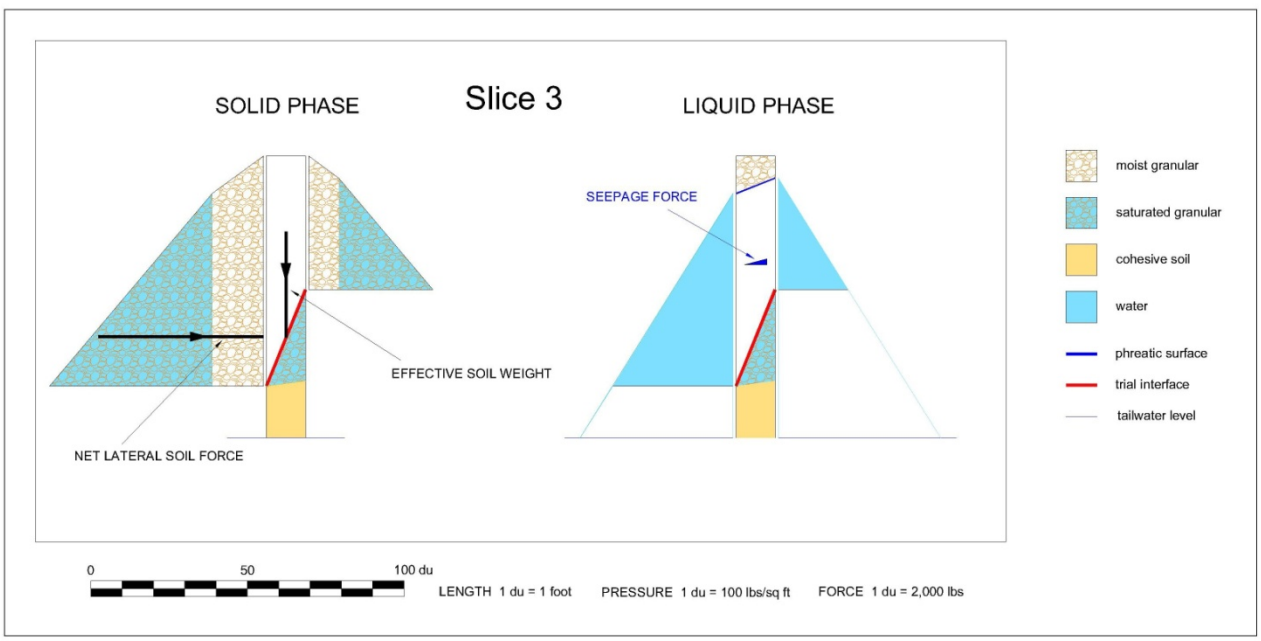

Figure 3

On the left side of Figure 3 is shown the resultant vertical force acting on the trial interface beneath Slice 3 drawn to scale, as is the resultant of the horizontal force brought about by lateral soil pressures on the two vertical sides of this slice. Similarly, Figure 4 shows the magnitudes of the same forces for Slice 8.

Figure 5 shows the force polygons for Slices 3 and 8 as derived from Figures 3 and 4. These lateral forces are of no consequence to stability since over the length of the stability section these obviously balance out, and all that remains is the difference in hydraulic/hydrostatic force between the first and last slices. They are only drawn here to indicate the relative magnitudes of the forces involved, and also make it apparent that they have not been ignored.

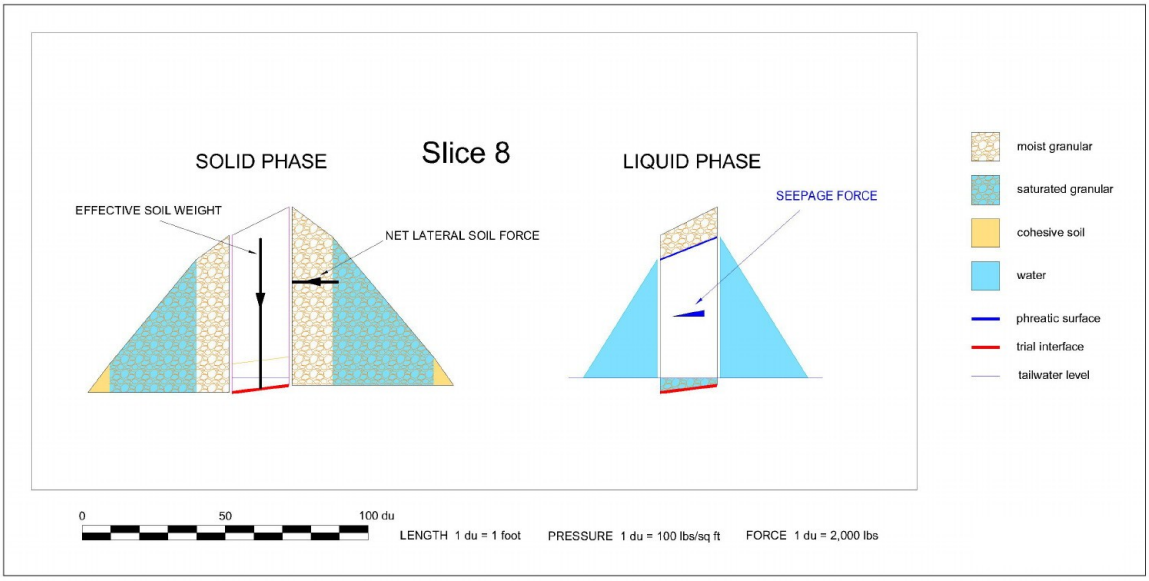

Figure 4

The two force magnitudes of interest here are:

P the component of the effective soil weight (W') parallel to the base of the slice, from which "H" is found, the force which depends upon the inclination of the interface, and

S the ultimate shear resistance to translation along the base.

These two pieces of information from each of the slices, together with the knowledge that inter-slice lateral forces cancel out, is all that is relevant to stability from the solid phase geometry.

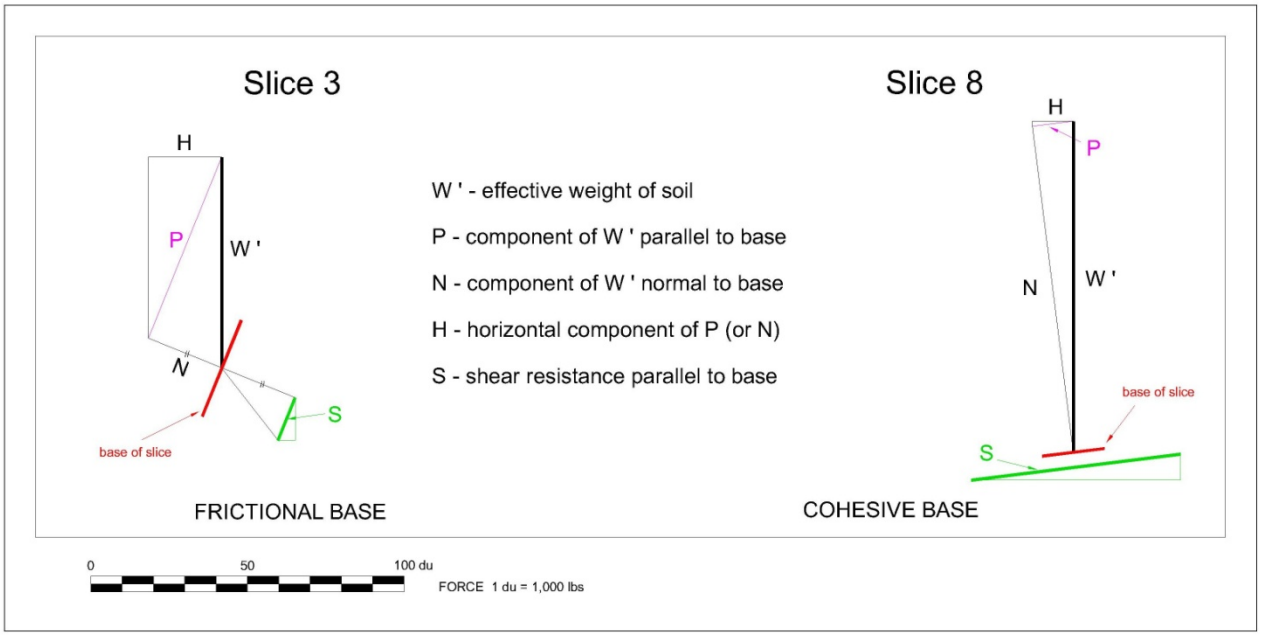

Figure 5

The methodology underlying this analytical approach is to treat the mass above the trial interface as a "free-body", that is to say, only boundary restraints to movement, and forces emanating from within the body itself need be considered in order to establish its stability with respect to translation along the interface.

Now that the solid phase has been accounted for, next the liquid phase must be considered. The fact that steady state seepage is occurring within the soil-structure, a persistent event which leads to energy being expended within the system, is a reality which must next be granted the particular attention and consideration it demands.

SEEPAGE FORCES

The essential starting point in appreciation Seepage Forces [**SF**] is the realization that pore water pressures, when quiescent/hydrostatic, can have no influence whatever on the buoyant soil-structure: It is only pore pressure gradients that effect the soil-structure, and thereby a slope's stability. The inclination of the phreatic surface is simply the maximum hydraulic gradient prevailing within the saturated soils beneath. As our attention moves down through the sub-parallel flow lines lower down the slice, the value of the gradient decreases gradually, until becoming zero at tailwater level.

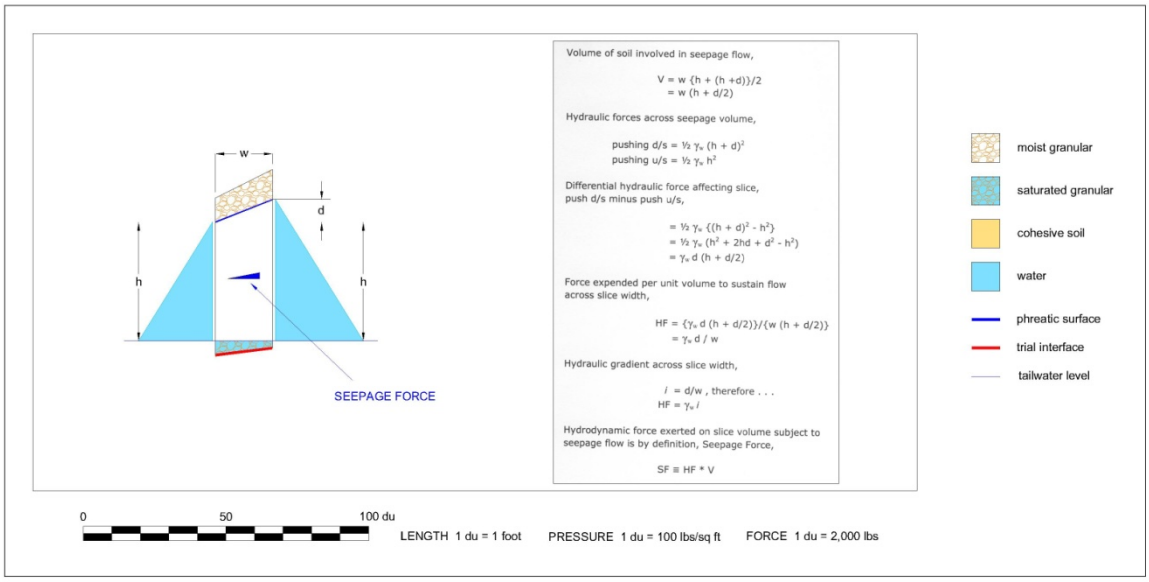

Figure 6

In Figure 6 we can see how a dissipating phreatic surface may be interpreted and translated into SF, such as those responsible for the vertical slopes in Figure 1. The diagram on the left of Figure 6 shows the hydraulic conditions that prevail in the liquid phase, using Slice 8 of Figure 2 as a typical case in point. The box to the right shows the relationship between the volume of the saturated soil within the slice (V), and the water pressures encompassing it, can be expressed as a single hydraulic vector effecting that particular slice. What Taylor called seepage force (Ref. 5)

We may now write that equation quantifying the SF acting within any slice in terms of variables we know:

$$SF = i \ \gamma_w \ V \qquad \text{Equation 1}$$

Where the first term is the average hydraulic gradient [***i***] prevailing within the saturated soils; γ_w is the unit weight of water, and V is the volume (area of unit thickness) subject to flow.

What may require some further elucidation is why, and how, SF could become attached to, and effect, the soil-structure phase. And that is explained as follows: In a porous medium such as earthfill, if there is an inclination to the phreatic surface, water will flow in the direction of the maximum downward gradient (flow line). The rate of flow will be in accordance with D'Arcy's Law, Q = *i k* V where *k* (permeability or hydraulic conductivity) is the resistance to flow, a measure of the energy requirements to sustain that level of flow.

Now switching attention from the perspective of the water phase to the soil-structure, that is, from energy expended to work done, the question arises: What is the work done on? And this is where Soil Mechanics' sister discipline, Fluid Mechanics, gives the answer. The work is done in overcoming the hydrodynamic Drag Forces which resists the relative motion of water past the solids. Drag is proportional to the square of the relative velocity, and is composed of frictional and viscous elements. For a detailed quantification of this exchange of energy see Ref 3 and 4**.**

Relevance of Seepage to Stability

So, what happens during steady state seepage through the porous media forming an embankment is that the water, in seeking tailwater level, its minimum potential, tends to drag the soil-structure in the same direction. In doing so it transfers by frictional and viscous drag forces, energy to the soil-structure, and this, as an intergranular compression parallel to the flow lines. And this would cause consolidation if the material were susceptible. This give-and-take between the fluid phase (seepage water) and the solid phase (soil-structure), from the point of view of Stability Analysis [**SA**], has two effects:

SF_H the horizontal component of SF is a destabilizing influence.

SF_V the vertical component of SF, can be a stabilizing influence in that it adds to the effective normal force on a sandy trial sliding interface, as the shear resistance in granular soils is increased, but on the other hand, brings no similar benefit to soil sections where the base soils are cohesive.

In Figure 6 the orthogonal sides of the solid blue triangle represent the horizontal (SF_H) and vertical (SF_V) components of SF to scale: The hypotenuse is centered on the point of action. So now return to the diagrams on the right side of Figures 3 and 4 it can be appreciated what they portray.

DEFINITION of FACTOR OF SAFETY for the h-Method

As is appropriate for the non-circular trial failure interface being considered here, the Factor of Safety [**FoS**] has been defined in terms of the ratio of horizontal forces. These being the ratio of shear forces resisting horizontal movement of the mass above the trial interface, to those soil-structure and seepage forces tending to cause such a movement.

In this formulation the numerator involves the standard equation for shear strength $s = c + (\sigma_n - u) \tan \varphi$, wherein the variables are expressed in scalar terms, as values of stress or pressure. Here, these quantities are converted to forces so that the resulting items, being vectors, can be examined for closure of their force polygons.

There are two forces which contribute to the frictional resistance, these being: the effective weight of the soil column, and then, the vertical component of the Seepage Force. Similarly, the denominator is comprised of the slice forces acting to promote lateral displacement. These are the horizontal components of the Seepage Force (SF_H) and H, the horizontal component of the normal force (N). In addition to these slice forces, in some situations there can be a simple single hydrostatic influence (L) introduced into the ratio-balance by standing water at either end of the section, such as the pond or a crack on the upstream side, &/or tailwater on the d/s side.

The Factor of Safety by this h-Method is therefore calculated as being the product of the following equation:

$$\frac{\Sigma \left[C_H + (W' + SF_V) \tan \varphi \cos^2 \beta \right]}{L + \Sigma \left[SF_H + H \right]} \qquad \text{Equation 2}$$

where, for each slice:

C_H	cohesive resistance, horizontal component
W'	effective weight of soil column
SF_V	Seepage Force, vertical component
φ	angle of frictional resistance
β	angular inclination of base
L	differential hydrostatic end forces, top minus bottom
SF_H	Seepage Force, horizontal component
H	horizontal component of normal force (N)
Σ	summation of values across all slices

Comparison of effects of Seepage on FoS

The geometry of the nominal embankment section to be examined has been depicted in Figure 2. Therein two options were left open: the location of the phreatic surface and the material types above and below the line of demarcation X – X. Now those options have been declared as shown in Figures 7 and 8.

The phreatic surface shown in Figure 7 is intended to represent the most liberal design approach by providing no drainage control of seepage flow. As a direct means of comparison, Figure 8 shows perhaps the most conservative of drainage designs, that being a chimney drain depressurize by a toe drain at about tailwater level.

There are only two types of soil considered: above the line is invariable a granular material which relies upon friction for its shear strength. Then, on alternate computations the material below X – X is made cohesive to ascertain to what degree impervious soils effect the results.

This arrangement yields eight different FoS as follows: 2 phreatic surface .x. two foundation materials .x. seepage force component being turned on or off.

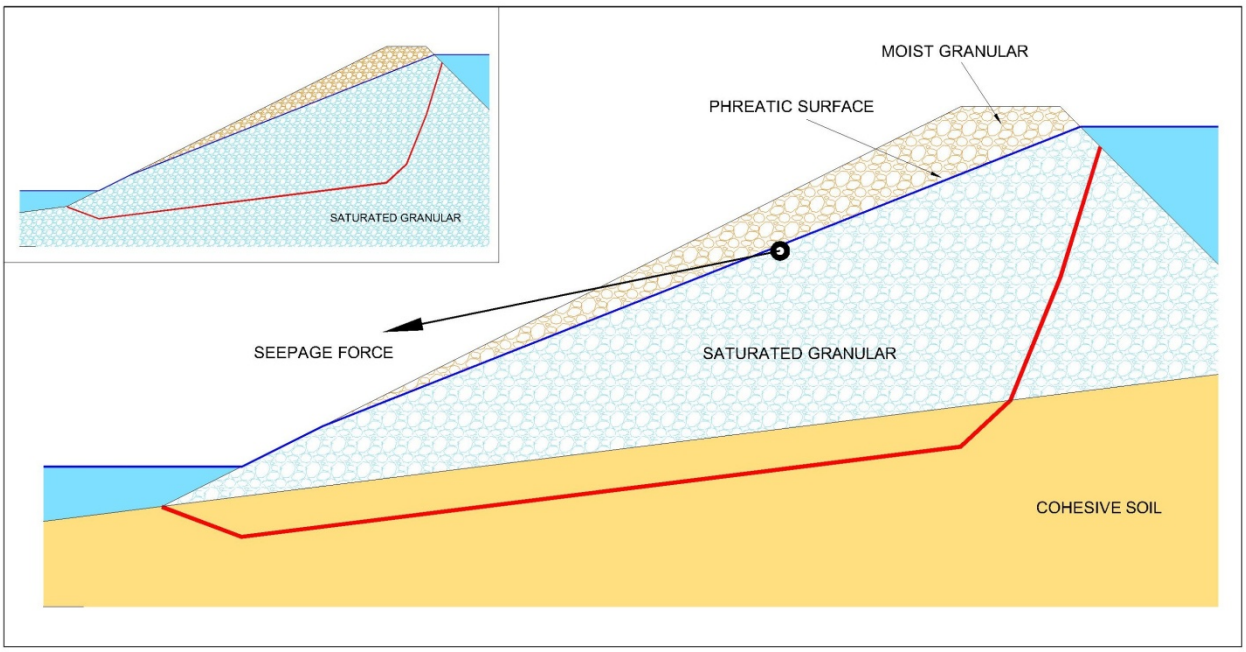

Figure 7

In both Figure 7 and Figure 8 the inclined arrow represents the resultant Seepage Force to scale, where each drawing unit represents 2,000 lbs force. The tail end of this vector is shown at a black donut which indicates the position of the centre of gravity of the sliding mass (effective weigh) is located.

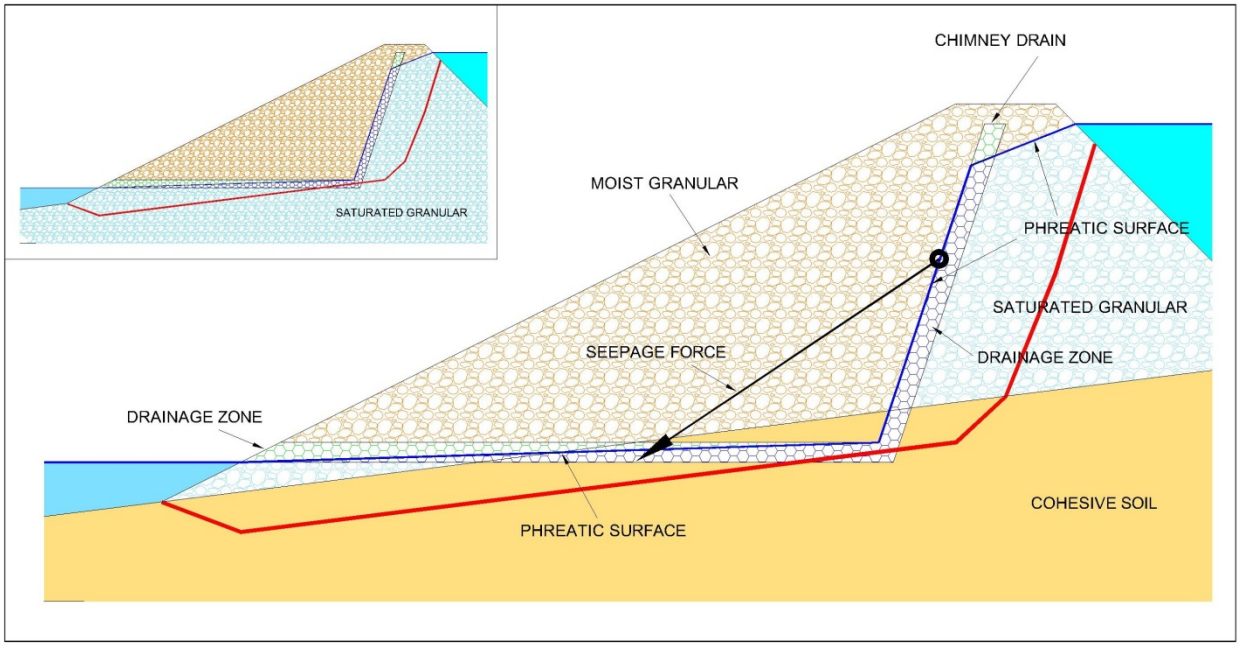

Figure 8

Results

The h-Method of determining the FoS was employed in each of those eight situations to compute its particular value: the results are tabulated in Table 1.

TABLE 1 – Numerical Results of Stability Analyses

A	B	C	D	E	F	G
Conditions		**Seepage Forces, kips**		**Factors of Safety**		**Error: Over-estimation of FoS**
Drainage	**Base soil**	**Horizontal**	**Vertical**	**h-Method**	**Zero SFs**	
None	Cohesive	199	41	1.937	4.135	114 % 2.14 : 1
None	Frictional			1.443	3.043	111% 2.11 : 1
Chimney	Cohesive	152	102	1.985	3.410	72 % 1.72 : 1
Chimney	Frictional			2.009	3.421	70 % 1.70 : 1

In Table 1 the seven column headings stand for:

A	the drainage design
B	the soil type through which the lower trial failure surface passes
C	the computed value of SF_H in units of 1,000 lbs
D	the computed value of SF_V in units of 1,000 lbs
E	FoS computed by the h-Method with the SFs included in the calculations
F	FoS computed by the h-Method with the SFs ignored
G	upper value = F*100/E , and lower value is ratio of F/E : 1

Discussion of Results

As explained earlier, the setup consciously set about preconditioning the example structure in order to reduce as much as possible the influence of resisting soil types, earthfill geometry and differential head upon the four different categories of embankment dams examined. Essentially, it then came down to what degree the positions of the phreatic surface, and whether the majority of the trial failure surface was granular or cohesive, to determine the slopes Factor of Safety. Columns A and B of Table 1 identify these four cases.

Column E lists the Factor of Safety for each of these situations as determined by the h-Method advocated here, and which incorporates variables designed to caters to seepage forces. Column F, again using the h-Method, but *turning off* its seepage force ability, lists the numbers which are produce when only hydrostatic fluid forces are considered to apply. Consequently, the differences between E numbers and those of F are believed to be as close to a true measure of the effect ignoring steady state seepage has on the numerical value attributed to the Factor of Safety assigned by the designers of on a water retaining earthfill.

It may be seen that the values for the two foundation soil type, as shown in Figure 7, where there is no provision (such as drainage in the downstream shell) to impede the full expression of its potential, there is a significant difference for the foundation types. This is because the high phreatic surface means that most of the overburden is buoyant and while this has little impact on the resistance of the clay base, it results in a large strength reduction in the case of the gravel base because W' is much reduced. Also, it may be seen in Table 1, that error in neglecting the existence of seepage, leads to an overestimation of safety of more than twice as much.

The second pairing is for the low, artificially depressed, phreatic surface shown in Figure 8. The results from the h-Method are both the same at 2.0. And in the case where seepage is ignored the computation, again, shows both values at 3.4, and overestimation of 70%.

The clay base h-Method result for the "chimney" drain is basically the same as for the same foundation without any provision seepage control. This is somewhat puzzling.

The above comparisons show that treating the seepage water within a slope as a matter of hydrodynamics, which it truly is, yields FoS values far lower than those which are arrived at when seepage is dealt with as a hydrostatic condition. In consequence, it must be concluded that our standard formulations of stability analyses which deal in terms of pore water pressures, rather than seepage forces, errs significantly on the unsafe side.

CONCLUDING REMARKS

There is a demonstrable need, as shown above, to upgrade the way in which the geotechnical community currently assesses the stability of natural slopes and earthfills subject to seepage flow, such as embankment dams or tailings dams.

The small group of men, who in the mid-nineteen hundreds, announced the advent of *Geotechnical Engineering* did no favor to Civil Engineering practice when they decided to concentrate on Soil Mechanics to the virtual exclusion of its sister discipline, Fluid Mechanics. The hydrodynamics inherent in Fluid Mechanics was lost in the process. This article is an attempt to undo that damage.

It is a mistake to treat saturated soil as a fixed entity. Such, it is not. Saturated soil is a 2-phase material where the two phases are not locked together, they do not necessarily act in concert. The water is provisionally free to move within the soil-structure, but can only avail itself of this freedom of relocation when/while it has the energy to do so. Water must expend energy to do the work entailed in overcoming the drag resistance that the soil-structure puts in the way of such relative motion between the phases.

Seepage Force is a convenient way to quantify the particular work-energy equation in governance here. Where there is no hydraulic gradient the situation is hydrostatic: the water cannot move. Therefore, other than by buoyant effect, the pore water has no power to move, nor to affect the soil-structure. On the other hand, the pore water beneath the phreatic surface, (the locus of *epwp* with reference to tailwater pressure, see Ref. 3), is an entire zone of hydraulic gradients balanced to provide the energy required to maintain the water flow down the streamlines against the impediment of the soil-structure's resistance to being intruded (k, permeability) upon.

Tailings dams are particularly vulnerable to the fact of seepage forces because, unlike civil engineering structures, they often are "works in progress", with their eventual height being much more than originally hoped for. In consequence, any provision for drainage may be scaled down to the extent that they essentially conform to the "no drainage" configuration examined here. And, by ignoring seepage forces the designers could have, by using standard practice, overestimate the stability by more than twice its true value. Another factor which should cause concern among the regulators of these embankments is the fact that in regions where precipitation exceeds evaporation, conscientiously abandoned sites still remain vulnerable, especially at their abutments.

The numbers indicate that ignoring steady state seepage forces is a serious matter, and it is clearly the civic, and social responsibility, of the geotechnical profession to get it right because no one else understands, or appreciates, the seriousness of this field.

The clear truth of the current situation is that the various bureaucracies who "earn their living" by dictating to their subject communities the rigid rules by which professionals should conduct themselves, are the real malefactors, and in the best of all world, these essentially inexperienced cliques should be held in ridicule.

Final remark

After the failure of the Mount Polley tailings dam, during a casual visit to the offices of Imperial Mines in Vancouver, the President of the company, offered an overhead photograph of the dam's left abutment sometime prior to the tailings dam's failure. It showed clearly that the downstream side of the crest had standing water on its surface. As the failure subsequently was initiated at this spot, it is this author's opinion that this image is sufficient evidence to conclude that the failure at Mount Polly is fully explicable as simply due to seepage force being present in the absence of adequate drainage. Nevertheless, it is necessary to point out that strain incompatibility issues between the abutment ground and the tailings material could have been a contributory factor, is unknown.

A CASE IN POINT: FORT PECK DAM

The slide that seriously damaged the all but completed Fort Peck dam on September 21st 1938, is perhaps most easily explained as follows:

This 9,000 ft long embankment dam was constructed using the hydraulic-fill technique which entailed discharging pumped silty sand slurry along the centerline: the silt fraction was contained around the axis, while the sandier fraction was formed into relative flat confinement shells on either side. The upstream face was built at slopes varying from 3:1 to 5:1 (H:V), while the d/s shell was built at 8½:1, [see Ref. 7].

The proposition being advanced here is that the Seepage Forces attending the progress of the hydraulic-fill method provide a *sufficient* explanation for all that happened that day as the embankment reached/approached the right abutment, and without the need for any further hypothesis.

Up to this juncture, as the earthfill moved across the Missouri River the supernatant water from the pumped fill had the opportunity to seep away to both side slopes, *as well as* from in front of the advancing embankment construction. But once the embankment came up against the rock wall of the abutment * the frontal seepage route was abruptly denied/cut off.

Figure 9: the dam "opened like a gate hinged on the east abutment"

The evidence suggests the following failure mechanics:

Once face to face with the immobile rock of the right abutment, the now entrapped seepage front, triggered a *quick* condition in the surrounding sandy fill and turned that soil into a highly energized heavy fluid.

That concentrated hydrodynamic energy, needing to vent, and given the geometric constrains of the situation, initially took the easiest exit route - the upstream face. That side was both steeper and shorter, as well as having less resistant to shear displacement because of being partially inundated. The upstream toe was submerged since the reservoir was already being used to store some water.

The aftermath of the consequential embankment failure is shown in an aerial photograph taken shortly after the event; Figure 9. What is immediately obvious is that this appearance is not the typical geometric mess which normally follows a slope failure, where there is generally little to be seen in common with the pre-failure slope face. Here, large areas, remnants of the former geometrical aspect of the slope, have been preserved even after being wafted considerable distances.

What appears to have happened here is that this part of the upstream slope, quite literally, had the ground swept out from beneath it. The source of the energy propelling these translations can be accounted for by the seepage forces that existed in front of the advancing embankment construction. Given that momentum, it then looks as if the outer face just reclined upon the heavy fluid which had undermined it.

Because of the rather gentle/soft nature of the forces impelling the earthfill movements it was possible for large sections/slabs of the original face to survive intact; these being held together by intergranular stresses referred to as "apparent-cohesion". This phenomenon can be quite influential in such fine-grained soils once they are less than fully saturated, that is, when they exist/behave under the rules of 3-phase physics. How to quantify these stresses is explained in detail in Ref. 1. Foam trails suggest that the main mass of debris rotated clockwise after being pushed away from its original location, and that surface water was not involved in the movements.

The decision to repair the damage, and subsequently commission the dam, was certainly the right decision: Once the hydrodynamic cause of failure was spent, it was quite safe to proceed under relatively/essentially static conditions.

* The four dark circles towards the upper right of the picture are the Gate Control Structures which were founded in bedrock, and thus can be used to identify the western/outer limit of the rock face.

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APPENDIX

While on the subject of stability analyses this seems to be a suitable occasion to comment on the angle of friction and its numerical relationship to the friction function.

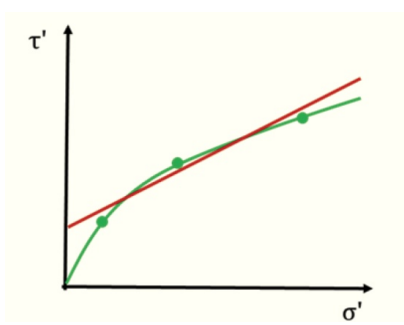
Since the function operating on phi is the tangent, it is important to realize that the numerical value of tan (ø) escalates rapidly as that parameter increases. Values of the tangent function at various ø angles are shown in the following tabulation:

Table 2

ø °	20	30	35	40	50	60	70	80
tan ø	0.36	0.58	0.70	0.84	1.19	1.73	2.75	5.67

Since the relationship between shear strength and effective overburden pressure is $\tau = \sigma' \tan ø$ it is important to keep in mind that the tangent function is far from proportionate: for instance when ø = zero, tan ø is also zero; when ø = 45° tan ø = 1; and if ø were to approach 90°, tan ø would approach infinity.

The following picture shows two ways (linear and modified) of presenting Mohr Coulomb failure criteria for a cohesionless soil. See Ref. 8.



These two lines, one straight the other curved, purport to represent the locus of the frictional component of the shear strength of a non-cohesive soil mass with respect to the effective normal stress exerted on the surface being considered.

The straight line is the statistical best *linear* fit to the three data points provided by laboratory testing of the soil under examination. This line is simply misleading because the linear regression analysis from which it is derived fails to take account of the most reliable data point available, and that is the origin of the axes, where it is known that when σ’ is zero so is τ.

Consequently, it is clear that the linear option should not be used in geotechnical practice since it does not at all represent the facts elicited from laboratory testing. Whereas, using as an example the curved line depicted above, shows that the friction component near the axes origin is 3.17 while at the top extremity of the curve it is only 0.28, that is a ratio of more than 11 to 1.

This shows that the value of the shear strength component of a granular soil is highly dependent on the effective overburden pressure. The clear implication to Stability Analysts is that the slope model ought to be subdivided into zones of σ’ so that appropriate friction angles may be attributed to the base of each slice along the failure surface.

END